PHENOMENOLOGICAL MODEL OF CREEP UNDER VARIABLE LOADS

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The quality of any theory of creep is evaluated by the degree to which its predictions of the behavior of a material agree with the behavior seem experimentally. The differences between different theories of creep (and their respective validities) are seen most clearly through the use of loading paths that might be referred to as "contrast" loadings. Examples of the latter are stepped loading, load reversal, and loading involving a sudden change in the type of stress state. Existing theories of creep are not able to predict many of the effects seen in regard to the behavior of materials subjected to contrast loading. As an illustration of this, we can point to the behavior of a material subjected to stepped loading with an increasing stress. The results of almost all published experimental data for such loading show that a stepped increase in stress produces a sharp increase in creep rate, with the order of magnitude of the rate being nearly the same as at the initial moment of creep.

Repeated attempts [1-4] have been made to describe the behavior of materials loaded with a stepped change in stress. However, none of these efforts have yielded results that adequately describe the empirical data.

In accordance with strain-hardening theory, when there is a stepped increase in stress (Fig. 1) at the moment of time  $t_x$  from  $\sigma_1$  to  $\sigma_2$ , section A"B of the creep curve - corresponding to the stress  $\sigma_2$  - is shifted along the x-axis until points A" and A' coincide. In this case, the experimental points are located above curve A'B' if the stresses do not exceed the elastic limit [5], i.e., the strain-hardening hypothesis produces a value that is lower than the experimental results.

This outcome led to the idea of moving segment AB, rather than segment A"B, to point A' (until point A coincides with point A'). This would in turn allow the creep strain p to be replaced as the strain-hardening measure by the parameter  $q = \int \sigma dp$  [1]. The variant of governing equations proposed in [3] leads to a similar restructuring of the creep curves for a stepped change in stress. Thus, the correspondence between the results in [3] and existing empirical data is about the same as in [1]. Neither variant is valid at stresses in excess of the elastic limit, since even strain-hardening theory gives an exaggerated result compared to the experiments [5]. The variants just referred to, moreover, predict strains even greater than the strains obtained from strain-hardening theory. Finally, the theoretical and experimental values also differ on creep rate, due to the above-mentioned stepped increase in stress.

A desire to be able to predict the behavior of a material subjected to contrast loading led to the development of the model described below. It is understood that this model is only one of several possible variants.

We will examine creep taking place with a stepped increase in stress. Figure 2 shows three creep curves for constant stresses  $\sigma_0$ ,  $\sigma_1$ , and  $\sigma_2$ , respectively. Let creep strain be accumulated in the specimen with a constant tensile stress  $\sigma_0$ . The stress suddenly increases to  $\sigma_1$  at the moment of time t =  $\vartheta_0$ , the specimen then undergoing further deformation. Later, at the moment of time t =  $\vartheta_1$ , the stress again increases suddenly to  $\sigma_2$ . The specimen undergoes further deformation at this stress.

Creep occurs at the stress  $\sigma_0$  by shear in certain slip systems (any other deformation mechanism can also be considered). After the stress is increased to  $\sigma_1$ , shear continues in these slip systems, and new slip systems that did not participate in creep at the stress  $\sigma_0$  become active. A similar situation unfolds with the subsequent increase in stress from  $\sigma_1$  to  $\sigma_2$ . This interpretation of events occurring in the material is obviously independent of the specific mechanism of deformation.

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Phenomenologically speaking, the proposed process can be realized by different means. As one possibility, let us assume that the activation of new slip systems with an increase in stress from  $\sigma_0$  to  $\sigma_1$  is evaluated on the basis of an amount of strain equal to the difference between the creep strains accumulated from the initial moment of time under the stresses  $\sigma_1$  and  $\sigma_0$ , respectively. This scheme is represented by the regions hatched with dashed vertical lines in Fig. 2. The strain is added to the strain which continues to accumulate at the stress  $\sigma = \sigma_0$ . Thus, at t >  $\vartheta_0$ 

$$p = p(t, \sigma_0) + [p(t - \vartheta_0, \sigma_1) - p(t - \vartheta_0, \sigma_0)].$$
(1)

Similarly, at  $t \geq \vartheta_1$ 

$$p = p(t, \sigma_0) + [p(t - \vartheta_0, \sigma_1) - p(t - \vartheta_0, \sigma_0)] +$$

$$+ [p(t - \vartheta_1, \sigma_2) - p(t - \vartheta_1, \sigma_1)].$$
(2)

The term in the last set of brackets in (2) corresponds to the region in Fig. 2 hatched with dot-dash lines.

The construction of creep curves for a stepwise-increasing stress does not require the use of constitutive equations if we have the creep curves for constant stresses corresponding to the stresses in the loading steps. If we lack this set of curves, then some form of constitutive relations will be needed. For example, proceeding on the basis of the strainhardening hypothesis

 $\dot{p}p^{\alpha} = a\sigma^{n}$ 

( $\alpha$ , a, and n being material constants at a fixed temperature, with the dot denoting differentiation with respect to time), we represent Eqs. (1) and (2) in the form

$$p = \left(\frac{at}{m} \sigma_0^n\right)^m, \quad (\alpha + 1) \ m = 1, \quad t \leq \vartheta_0,$$
$$p = \left(\frac{at}{m} \sigma_0^n\right)^m + \left[\frac{a \left(t - \vartheta_0\right)}{m}\right]^m \left(\sigma_1^{nm} - \sigma_0^{nm}\right), \quad \vartheta_0 \leq t \leq \vartheta_i$$

$$p = \left(\frac{at}{m}\sigma_0^n\right)^m + \left[\frac{a\left(t-\vartheta_0\right)}{m}\right]^m \left(\sigma_1^{nm} - \sigma_0^{nm}\right) + \left[\frac{a\left(t-\vartheta_1\right)}{m}\right]^m \left(\sigma_2^{nm} - \sigma_1^{nm}\right), \quad t \ge \vartheta_1.$$
(3)

It is evident that, in accordance with the empirical data, when the stress is increased the creep rate will be the same as at the initial moment of creep, i.e. it will be infinite. Creep rate subsequently decreases and approaches the value corresponding to creep at the new constant stress.

Creep curves calculated by the proposed method with a stepwise-increasing stress were compared with experimental curves obtained for aluminum alloys Dl6AT at 150°C [6] and Dl6T at 150°C [7]. Figure 3 shows sample comparisons for Dl6AT at 150°C. The theoretical creep curves are represented by lines, while the numbers denote stress values.

We should point out that we looked at experiments in which the stresses did not exceed the elastic limit, since creep behavior is quite different at greater stresses [5].

An analysis of the results shows that the creep curves calculated using the proposed model agree fairly well with the experimental curves. The largest deviations remain within the scatter of the empirical data. At the same time, we reiterate that, in contrast to other models, the given creep model ensures agreement between the theoretical and experimental creep rates at the moment of stress change. The limiting values of creep rate are also the same in each case.

Now let us turn to the case of creep when the stress change is suddenly reversed after deformation at the constant stress  $\sigma_0$  for the period of time  $\vartheta_0$ . Then, after an interval of time equal to  $\vartheta_1 - \vartheta_0$  (recovery), the stress change is reinstated and creep continues at the stress  $\sigma_0$  (see Fig. 2). During the recovery period  $(\vartheta_0 \le t \le \vartheta_1)$ , the strain recovers by the amount  $p_r$  (the recovery region is represented by vertical lines with cross hatches in Fig. 2). The actual creep curve on this section will be A'B<sub>0</sub> if we assume that restoration of the stress will be followed not only by the accumulation of strain in accordance with the creep curve when  $\sigma = \sigma_0$  (with section AB of the curve being shifted parallel to the x-axis to the position A'B'), but also by restoration of the recovery strain  $p_r$  (see Fig. 2).

Thus, an explanation has been found for the empirically observed abrupt increase in creep rate at the moment of restoration of the stress. It is evident that, in accordance with the proposed model, over time the creep rate will assume the value that would have been attained with a constant stress  $\sigma = \sigma_0$ . This is the result seen in experiments.

Thus, the creep strain is as follows during the recovery period  $(\vartheta_0 \leq t \leq \vartheta_1)$ 

$$p = p(\sigma_0, \vartheta_0) - p_r(\sigma_0, t - \vartheta_0).$$
(4)

while after recovery  $(t \ge \vartheta_1)$  (see Fig. 2) it is equal to

$$p = p(\sigma_0, t - \vartheta_1 + \vartheta_0 - \vartheta_*) + p_r(\sigma_0, t - \vartheta_1).$$
<sup>(5)</sup>

If the required set of creep curves for constant stresses and recovery is available, then creep curves for step loading with recovery can be constructed from Eqs. (4) and (5) without the use of constitutive relations. Lacking this set of curves, the use of some kind of creep hypothesis (such as the strain-hardening hypothesis) will be necessary. Equation (3) is valid at  $t < \vartheta_0$ , while at  $\vartheta_0 \le t \le \vartheta_1$ 

$$p = \left(\frac{a\vartheta_0}{m}\,\sigma_0^n\right)^m \to \left[\frac{A\left(t-\vartheta_0\right)}{M}\right]_{\cdot}^M \left[\left(\frac{a\vartheta_0}{m}\,\sigma_0^n\right)^m\right]^{bM}\,\sigma_0^{NM},$$

At t  $\geq \vartheta_1$ 

$$p = \left[ \left\{ \left( \frac{a\vartheta_0}{m} \sigma_0^n \right)^m - \left[ \frac{A \left( \vartheta_1 - \vartheta_0 \right)}{M} \right]^M \left[ \left( \frac{a\vartheta_0}{m} \sigma_0^n \right)^m \right]^{bM} \sigma_0^{NM} \right\}^{1/m} + \frac{a \left( t - \vartheta_1 \right)}{m} \sigma_0^n \right]^m + \left[ \frac{A \left( t - \vartheta_1 \right)}{M} \right]^M \left[ \left( \frac{a\vartheta_0}{m} \sigma_0^n \right)^m \right]^{bM} \sigma_0^{NM},$$
$$M(\beta + 1) = 1.$$

Here, we made use of the recovery law [8]

$$\dot{p}_r |p_r|^{\beta} = A p_0^b (\sigma_0 - \sigma_r)^N, \tag{6}$$

in which  $p_0$  is the creep strain accumulated at the stress  $\sigma_0$  up to the moment when the stress is reduced from  $\sigma_0$  to  $\sigma_r$ ; A,  $\beta$ , b, and N are material constants at a fixed temperature.

If the stress is not completely removed (partial recovery), i.e., if it is suddenly partially reduced from  $\sigma_0$  (after creep at  $\sigma_0$  for the period of time  $\vartheta_0$ ) to  $\sigma_*$ , then the creep process will consist of the accumulation of strain at the reduced stress ( $\sigma = \sigma_*$ ) and partial recovery [8]. The corresponding creep curve during partial recovery within the time interval  $\vartheta_1 - \vartheta_0$  and after return to the stress to  $\sigma_0$  ( $t \ge \vartheta_1$ ) is constructed similarly to the case of complete recovery ( $\sigma_* = 0$ ).

Having used the strain-hardening hypothesis, for  $\vartheta_0 \leq t \leq \vartheta_1$  we obtain

$$p = \left[\frac{a\vartheta_0}{m}\sigma_0^n + \frac{a(t-\vartheta_0)}{m}\sigma_*^n\right]^m - \left[\frac{A(t-\vartheta_0)}{M}\right]^M \left[\left(\frac{a\vartheta_0}{m}\sigma_0^n\right)^m\right]^{bM}(\sigma_0 - \sigma_*)^{NM},$$

while at  $t \geq \vartheta_1$ 

$$p = \left[ \left\{ \left[ \frac{a\vartheta_0}{m} \sigma_0^n + \frac{a\left(\vartheta_1 - \vartheta_0\right)}{m} \sigma_{\star}^n \right]^m - \left[ \frac{A\left(\vartheta_1 - \vartheta_0\right)}{M} \right]^M \left[ \left( \frac{a\vartheta_0}{m} \sigma_0^n \right)^m \right]^{bM} \times \right. \\ \left. \times \left( \sigma_0 - \sigma_{\star} \right)^{NM} \right\}^{1/m} + \frac{a\left(t - \vartheta_1\right)}{m} \sigma_0^n \right]^m + \left[ \frac{A\left(t - \vartheta_1\right)}{M} \right]^M \left[ \left( \frac{a\vartheta_0}{m} \sigma_0^n \right)^m \right]^{bM} \left( \sigma_0 - \sigma_{\star} \right)^{NM}.$$

Figure 4 shows sample creep curves calculated on the basis of the proposed model for the case of stepped loading, including complete and partial recovery. The calculations were performed for aluminum alloy D16AT at 150°C. These results are compared with experimental data from [6, 7]. The numbers represent stresses on different sections of the creep curve. We moved the origin of the curves (designating it with the number 0) to permit their more compact representation.

Analysis of the results shows that the creep curves calculated on the basis of the proposed model agree satisfactorily with the empirical data. The deviations of the theoretical curves from the experimental curves seen in certain cases do not exceed the scatter of the empirical data.

Along with recovery equation (6), we used the variant

$$p_r |p_r|^{\beta} = A p_0^b \left( \frac{\sigma_0 - \sigma_r}{\sigma_0} \right)^N,$$
(7)

in which the rate of recovery depends on the relative rather than the absolute stress reduction. The degree of correspondence between the creep curves obtained with Eq. (7) and the experimental data is the same as with the use of Eq. (6). However, no definitive conclusions can be made from this in regard to the establishment of any kind of recovery law or the derivation of such a law, since doing so would require a broader range of experimental data on recovery for different stress reductions and different preceding creep strains.

It should also be noted that the conclusions reached on the basis of the proposed model are independent of both the character of the recovery law and the mechanism of deformation. However, they do of course depend on the method used to account for the contribution to creep of slip systems or deformation systems activated in the course of deformation. It is understood that, in the absence of the requisite set of creep and recovery curves and the consequent need to use some specific creep and recovery hypotheses, the degree to which the given model will agree with existing empirical data will depend on the chosen hypotheses.

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## DISSIPATIVE MESOSCOPIC STRUCTURES IN

## PLASTIC DEFORMATION

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The studies [1-5] presented empirical proof of the existence of a new dissipative structure in thin specimens plastically deformed by shear under pressure. This so-called nonuniform-pressure structure (NPS) governs plastic motions at levels between the microscopic and macroscopic scales. Although there has been relatively little study of the processes which take place at these levels, information on them is needed to unify dislocation theory with the theory of plasticity of continua [6]. Also, events occurring at the mesoscopic level to a significant extent determine the course of friction and wear [3] and industrial processes such as mechanical fusion [4, 5].

In the present study, we examine processes involved in the creation, growth, relaxation, and plastic motion of high-pressure regions (HPR) in materials. Such regions are elements of the nonuniform-pressure structures mentioned above. We will also examine the conditions under which the latter structures are formed.

<u>Experimental Data</u>. Specimens 0.15-0.5 mm thick and 8-12 mm in diameter were compressed between flat parallel dies to pressures of 0.2-1.4 GPa and deformed plastically by rotation of one of the dies. Each specimen consisted either of a single metal or of three layers of two alternate metals (a sandwich). We used light microscopes and scanning and transmission (replica method) electron microscopes to study the surface of the specimens and the middle layers of the sandwiches (exposed by chemical dissolution of the outermost layers), as well as cross sections. The main findings are discussed below.

On the pre-oxidized zinc surfaces adjacent to the dies, the oxide films were displaced from some sections and concentrated in other sections (Fig. la,  $\times 60$ ). The nonuniformity of the distribution of these displacements over the contact surfaces increased with an increase in shear. The material of some sections was entrained by the dies and pulled onto adjacent sections in the form of thin wedges. The wedges were lifted above the surface after the load was removed (Fig. lb,  $\times 400$ ).

In the specimens of ultrapure aluminum, after deformation fragments of the oxide films ended up embedded in the specimen. Thus, fragments of aluminum oxide in the form of groups of long narrow streaks (Fig. 2a,  $\times 5000$ ) were observed on a replica from a surface exposed by layer-by-layer chemical dissolution of the specimen. When we etched the specimen in a preparation that acted selectively on the material containing oxides, deep pits that sometimes extended completely through the specimen were formed (Fig. 2, where b shows the surface of a cross section magnified 100 times and c shows the surface of the specimen magnified 170 times).

Plastic motions of the material were established from changes in the location of the oxide-film fragments (which served as reference points) and the formation of the above-

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